Math 250 – Sect.2.6: Related Rates

Review. Find each of the following:

a. $\frac{d}{dx}(5x^3)$  b. $\frac{d}{dt}(5x^3)$  c. $\frac{d}{dt}(5x^3 + y^2)$

*Both (b) and (c) require CHAIN RULE because the derivative is with respect to t, yet the expressions are not in terms of t.

Let's try another:

d. If $xy = 4$, find $\frac{dx}{dt}$ when $x = 8$ and $\frac{dy}{dt} = -2$.

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| RELATED RATES problems concern rates of change of two or more related variables that are changing with respect to *time*. |

For example: Suppose a point is moving along the graph of $y = \sqrt{x}$ in such a way that $\frac{dx}{dt} = -1\ cm/\sec$.
Find the rate at which the *y* coordinate is changing (i.e. find _________) at the moment when $x = 9$ cm.

**To solve Related Rates Problems, first list the following:**

**a. What you are GIVEN. This means the specific information listed in the problem.**

**b. What you are to FIND.**

**c. What you KNOW. This means any formulas (such as area, volume, Pythagorean Theorem, etc.) that would be appropriate for the situation. A picture might also help**

**THEN, check to see if the formula you wrote in the KNOW part relates the variables that you have information about in the GIVEN, and what you want to FIND. If so, differentiate the formula from the KNOW *with respect to time!***

**Try these.**

1. Bacteria are growing in a circular colony one bacterium thick. The bacteria are growing at a constant rate, thus making the area of the colony increase at a constant rate of 12 $mm^2/hr$. Determine how fast the radius of the circle is changing at the moment when it equals 3 *mm*.

GIVEN:

FIND:

KNOW:

Solve:

2. Johnny B. Good blows up a spherical balloon. In order for the radius to increase at 2 cm/sec, how fast must Johnny blow air into the balloon when r = 3 cm?

GIVEN:

FIND:

KNOW:

Solve:

3. A point is moving along the graph of $y = \sqrt{x}$ in such a way that $\frac{dx}{dt} = 0.5$ cm/sec. Find the rate at which the *distance* between the point and the origin is changing when $x = 4$.

GIVEN:

FIND:

KNOW:

Solve:

4. The edges of a cube are expanding at a rate of 1.5 cm/min. Find the rate at which the (a) volume, and (b) surface area are changing WHEN the edge is 2 cm long.

**a. For Volume:**

GIVEN:

FIND:

KNOW:

Solve:

**b. For the Surface Area:**

GIVEN:

FIND:

KNOW:

Solve:

5. A reservoir is in the shape of a cone, vertex down. The radius at the top is 20 meters, and the height of the reservoir is 15 meters. Water is pouring OUT of the reservoir at a rate of 2 $m^3/\text{min}$. Find the rate at which the HEIGHT of the water is changing at the moment when the height is 10 meters.

DRAW A PICTURE:

6. A 20 ft. ladder leans against the wall. The base of the ladder is being pushed TOWARD the wall at a rate of 0.2 ft/sec. Find:

a. The rate at which the top of the ladder is moving up the wall at the moment when the base of the ladder is 6 ft. from the wall.

b. The rate at which the AREA of the triangle formed by the wall, ladder, and ground is changing at the same moment as in part (a).